

Comparing methods for uncertainty propagation in LCA

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Background: why?

- Uncertainty analysis, such as Monte Carlo sampling (MCS) is time-consuming;
- There are other methods available that are claimed to be more efficient;
- No consistent comparison yet made in literature.

Matrix representation*

	Fuel production	Fish production		
Fuel (kg)	10	-4	Technology matrix: A	0
Fish (kg)	0	11		100
CO ₂ (kg)	9	3	Intervention matrix: B	?
SO ₂ (kg)	2	3		?
Crude oil (liter)	-12	0		?

Final demand vector: **f**

Environmental intervention vector: **g**

$$\mathbf{A} = \begin{pmatrix} 10 & -4 \\ 0 & 11 \end{pmatrix}$$

$$\mathbf{f} = \begin{pmatrix} 0 \\ 100 \end{pmatrix}$$

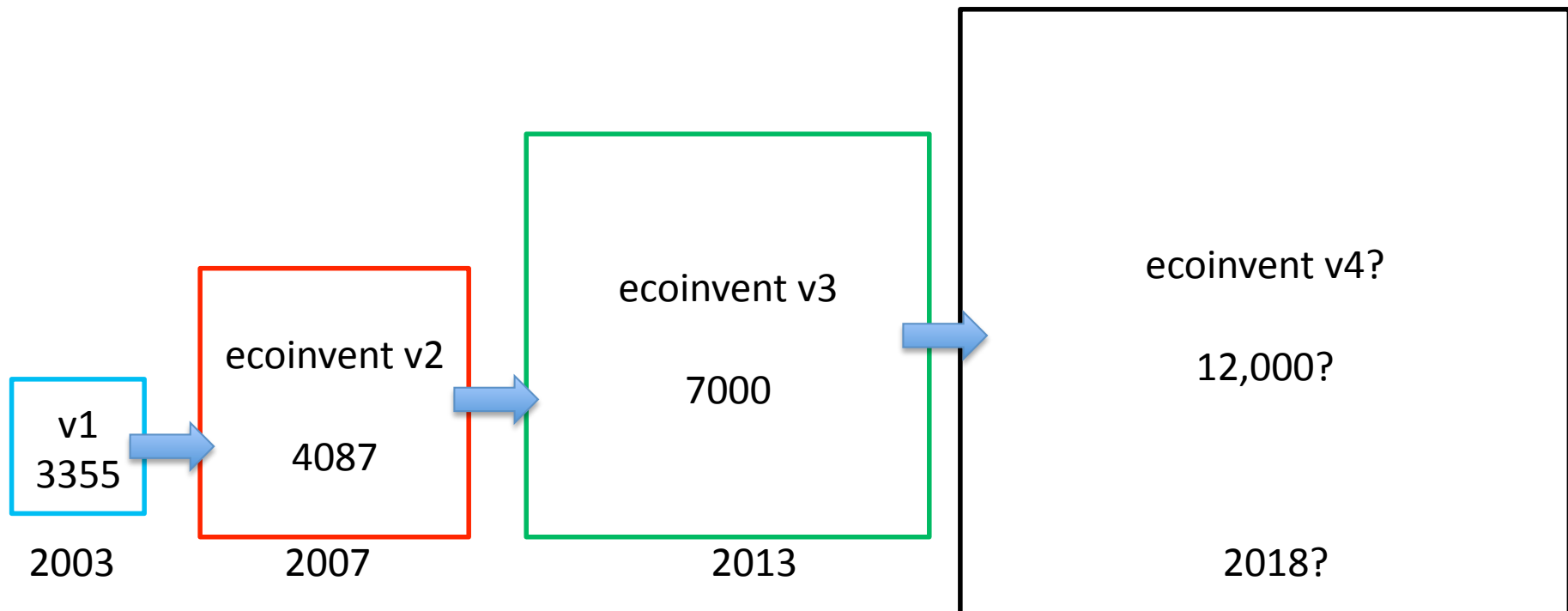
$$\mathbf{B} = \begin{pmatrix} 9 & 3 \\ 2 & 3 \\ -12 & 0 \end{pmatrix}$$

$$\mathbf{g} = \mathbf{B}\mathbf{A}^{-1}\mathbf{f} = \mathbf{B}\mathbf{s} = \frac{1}{11} \begin{pmatrix} 660 \\ 380 \\ -480 \end{pmatrix} \approx \begin{pmatrix} 60 \\ 34.55 \\ -43.64 \end{pmatrix}$$

Problem statement

Uncertainty analysis is time-consuming because:

- the A-matrix is large;
- calculation of the inverse is slow.



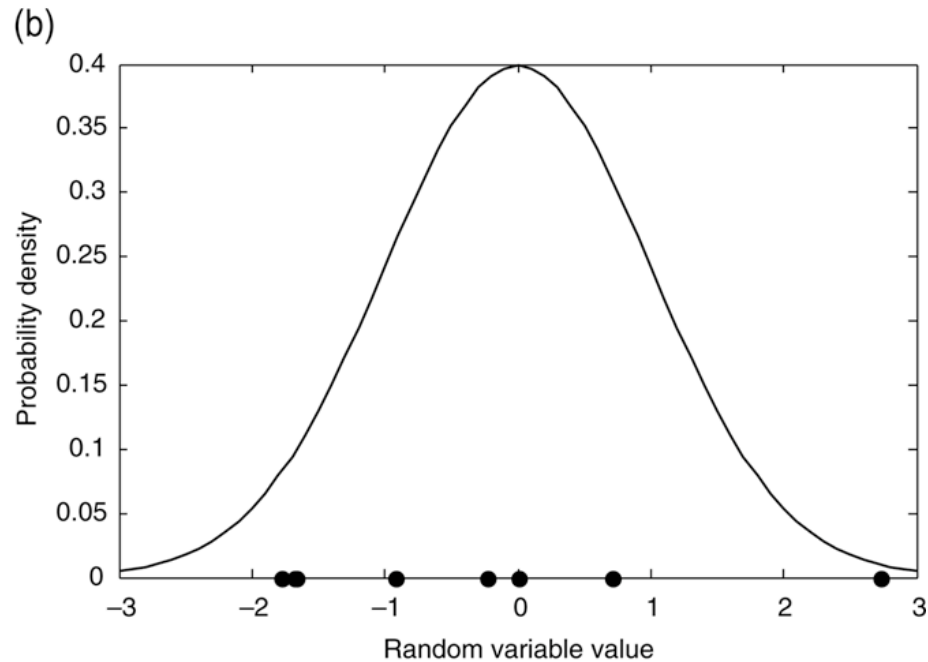
Reduce sampling

Default method Monte Carlo sampling (MCS) is compared to:

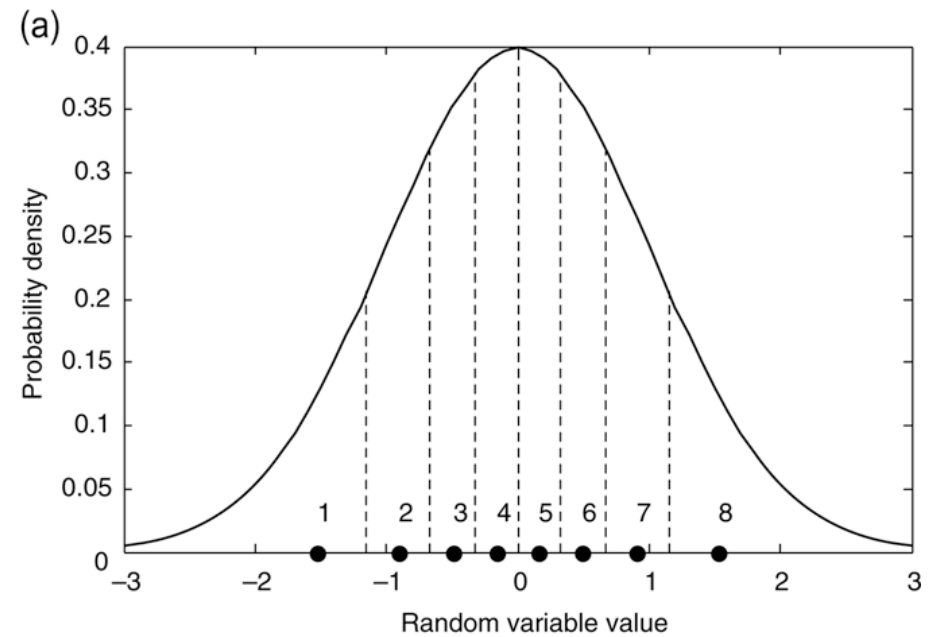
- Latin hypercube sampling (LHS)
- Fuzzy interval arithmetic (FIA)
- Analytical uncertainty propagation (AUP)

Sampling methods

1. MCS:



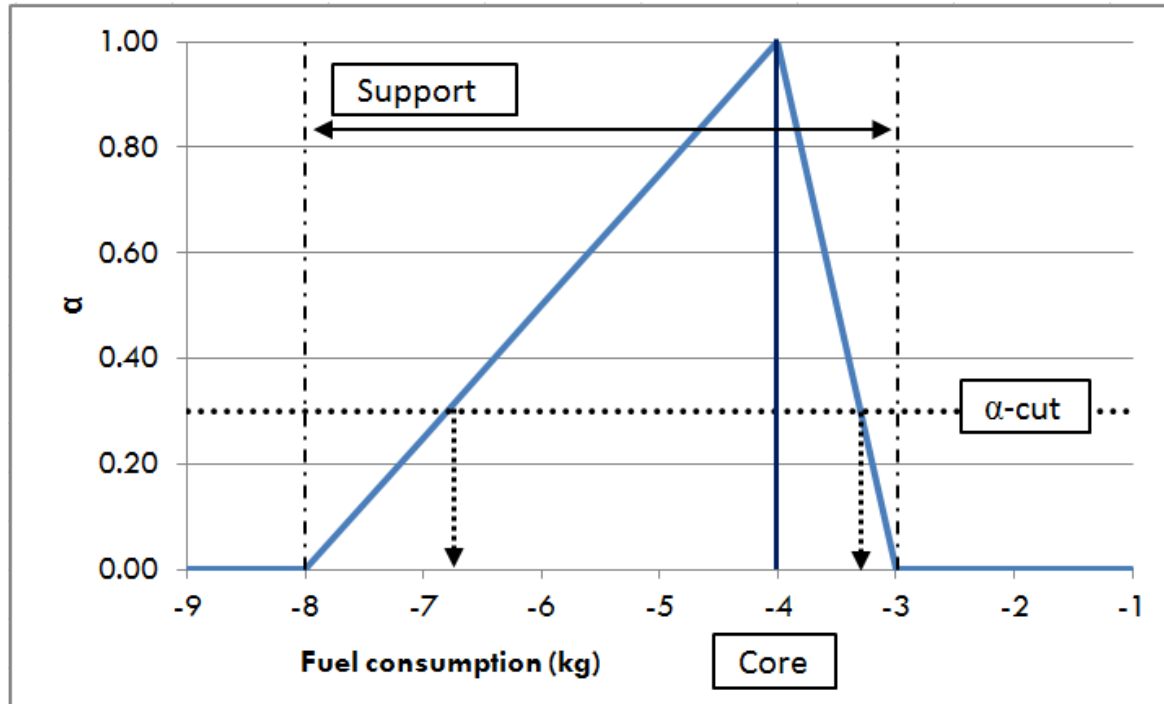
2. LHS:



Reference: Nikos D. Lagaros, et al (2010)
"Neurocomputing strategies for solving
reliability-robust design optimization
problems"

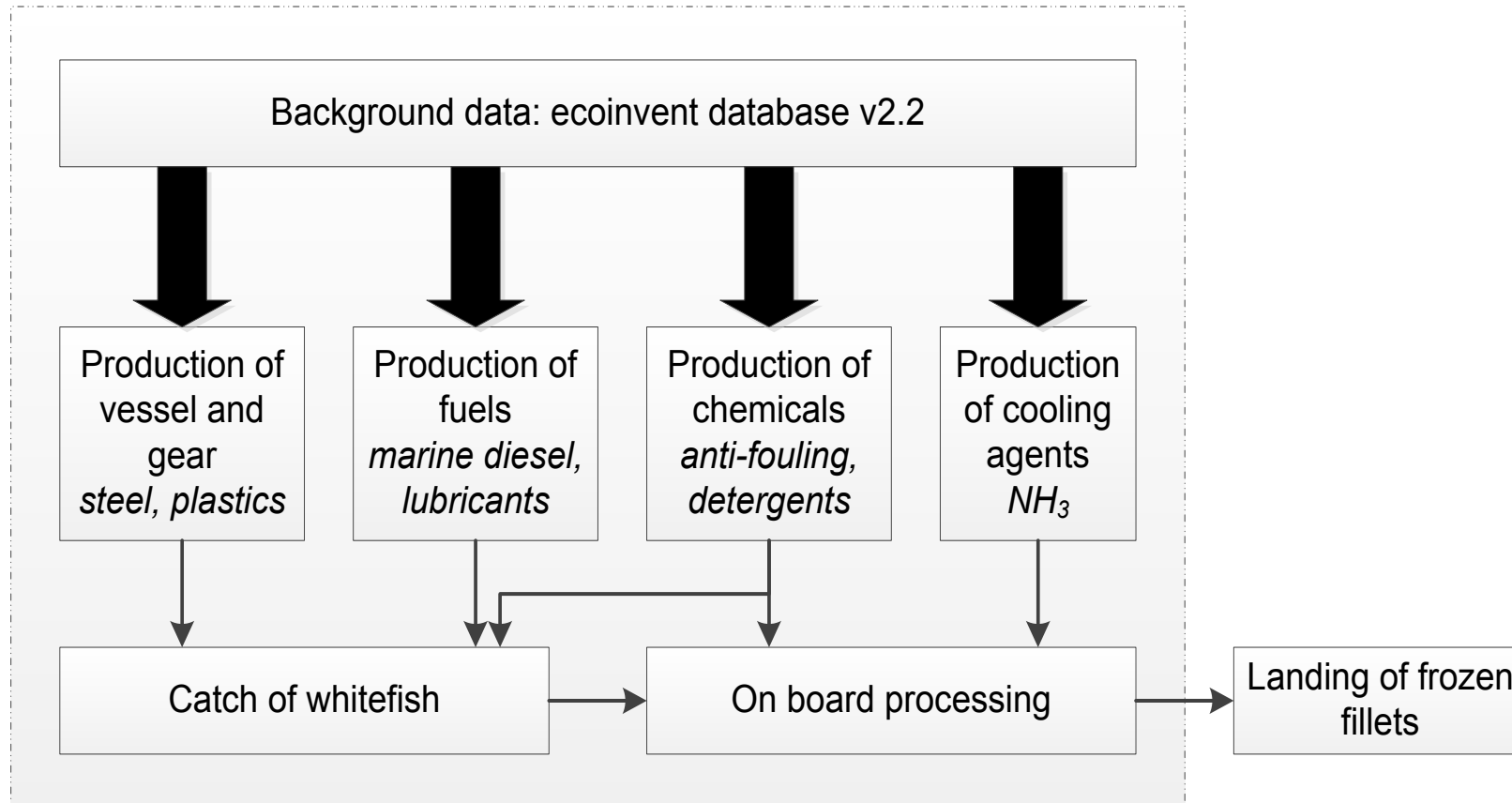
Analytical methods

3. FIA:



4. AUP: first order Taylor approximation

The case study



Results

	mean	sd	Calculation time	Memory usage	Inputs
MCS ₁₀	27.0 • 10 ⁵ kg CO2 eq/10 ⁶ kg fish	3.10	1.3 s	0.8 MB	Mean, parameter of dispersion, distribution function
MCS ₁₀₀	25.7	3.59	13 s	0.8 MB	idem
MCS ₁₀₀₀	25.8	3.49	2.1 min.	0.8 MB	idem
LHS ₁₀	26.2	4.14	1.9	5.8 MB	Mean, parameter of dispersion, distribution function
LHS ₁₀₀	25.9	3.29	19 s	53 MB	idem
LHS ₁₀₀₀	25.9	3.57	3 min.	523 MB	idem

	Core value	sd	range	Calculation time	Memory usage	Inputs
FIA ₁₁	27.0	n/a	20.5-30.6	5 s	8 MB	Core value, upper and lower bound, shape
AUP ₁	n/a	3.31	n/a	1.5 s	1 MB	Parameter of dispersion

Conclusions

- Combine different approaches;
- FIA works well with large uncertainties;
- Prefer MCS over LHS.

Thanks!

Acknowledgements:

